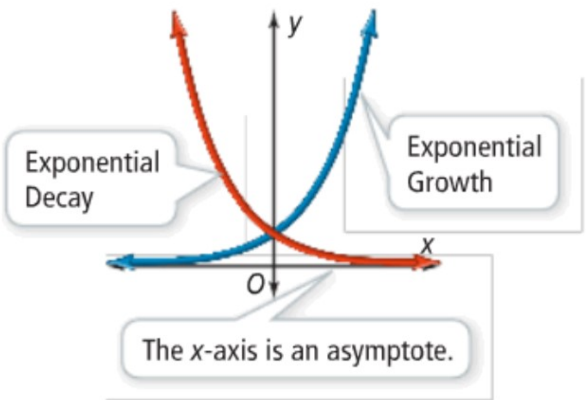
Exponential Models

Objective To model exponential growth and decay



Essential Understanding You can represent repeated multiplication with a function of the form $y = ab^x$ where b is a positive number other than 1.

An **exponential function** is a function with the general form $y = ab^x$, $a \neq 0$, with $b > 0$, and $b \neq 1$. In an exponential function, the base b is a constant. The exponent x is the independent variable with domain the set of real numbers.

Exponential Models

Time (hours)	Number of bacteria
0	1
0.5	2
1.0	4
1.5	8
2.0	16
2.5	32
3.0	64
3.5	128
4.0	256
4.5	512
5.0	1,024
5.5	2,048
6.0	4,096
6.5	8,192
7.0	16,384
7.5	32,768
8.0	65,536
8.5	131,072
9.0	262,144
9.5	524,288
10.0	1,048,576

(a)

Bacterial Decay

$$b = (150\,000\,000 \text{ bacteria})\left(\frac{1}{3}\right)^{\frac{t}{5\text{ days}}}$$

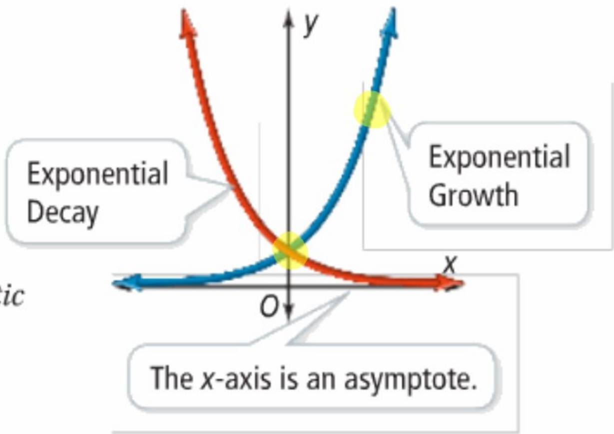
Number of Days	Number of Bacteria
1	120 000 000
2	96 700 000
3	77 600 000
4	62 300 000
5	50 000 000
6	40 100 000
7	32 200 000
8	25 900 000
9	20 800 000
10	16 700 000
11	13 400 000

Exponential Models

Two types of exponential behavior are *exponential growth* and *exponential decay*.

For **exponential growth**, as the value of x increases, the value of y increases. For **exponential decay**, as the value of x increases, the value of y decreases, approaching zero.

The exponential functions shown here are *asymptotic* to the x-axis. An **asymptote** is a line that a graph approaches as x or y increases in absolute value.



$$g(x) = ab^{x-h} + k \text{ where } b > 1$$

In a transformation, the point $(0, 1)$ becomes $(h, a + k)$ and $(1, b)$ becomes $(1 + h, ab + k)$. The asymptote $y = 0$ for the parent function becomes $y = k$.

Exponential Models

take note

Concept Summary Exponential Functions

For the function $y = ab^x$,

- if $a > 0$ and $b > 1$, the function represents exponential growth.
- if $a > 0$ and $0 < b < 1$, the function represents exponential decay.

In either case, the y-intercept is $(0, a)$, the domain is all real numbers, the asymptote is $y = 0$, and the range is $y > 0$.

$y = 3(2)^x$		$y = (2)^x \longrightarrow y = 1(2)^x$
All real #s	Domain	All real #s
$y > 0$	Range	$y > 0$
$(0, 3)$	y-intercept	$(0, 1)$
$y = 0$	asymptote	$y = 0$

Exponential Models

Identify each function or situation as an example of exponential growth or decay. What is the y-intercept?

A $y = 12(0.95)^x$

Since $0 < b < 1$, the function represents exponential decay. The y-intercept is $(0, a) = (0, 12)$.

B $y = 0.25(2)^x$

Since $b > 1$, the function represents exponential growth. The y-intercept is $(0, a) = (0, 0.25)$.

Exponential Models

Got It? **2.** Identify each function or situation as an example of exponential growth or decay. What is the y-intercept?

a. $y = 3(4^x)$

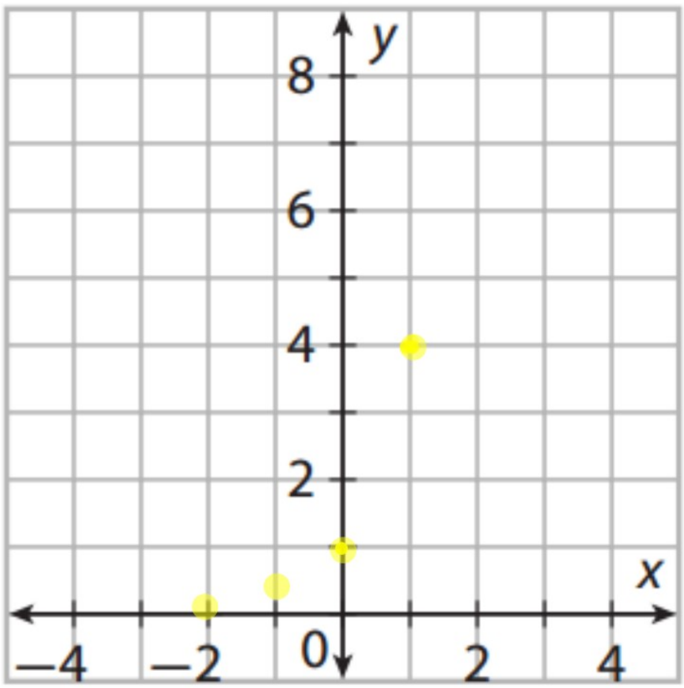
b. $y = 11(0.75^x)$

Domain
Range
y-intercept
asymptote

Exponential Models

What is the graph of $f(x) = 4^x$

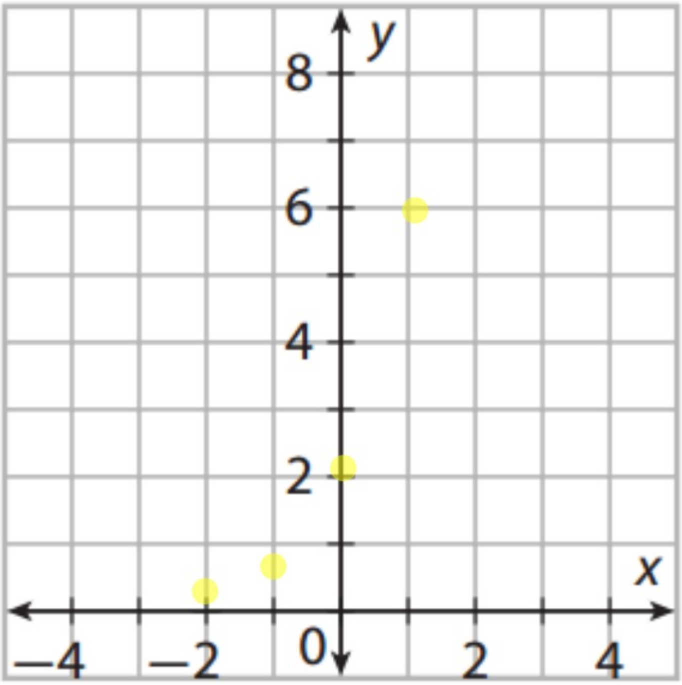
x	$f(x)$
-2	***0.0625***
-1	***0.25***
0	***1***
1	***4***
2	***16***



Exponential Models

What is the graph of $f(x) = 2(3)^x$

x	f(x)
-2	*2/9*
-1	*2/3*
0	*2*
1	*6*
2	*18*



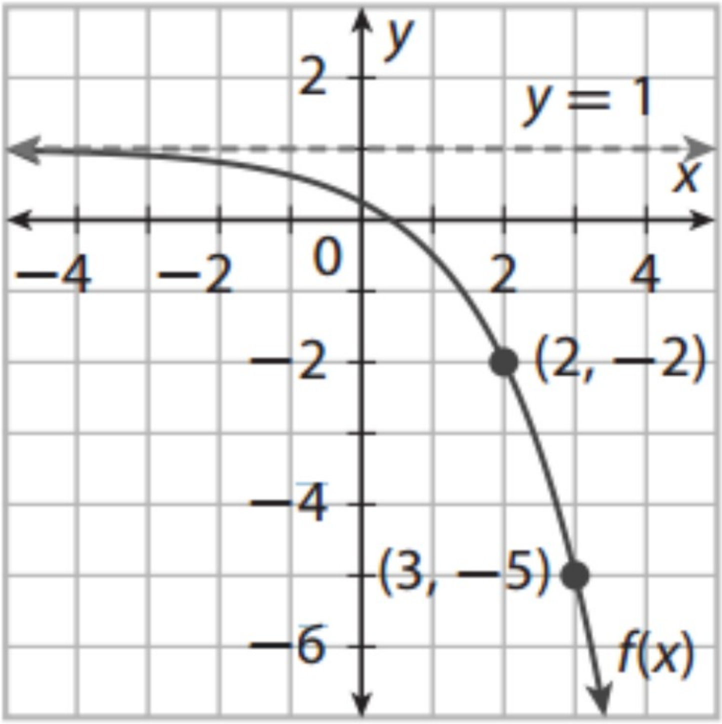
Exponential Models

Writing Equations for Combined Transformations $g(x) = a\left(b^{x-h}\right) + k$

$\left(h, a + k\right)$

$\left(1 + h, ab + k\right)$

$g(x) = -3\left(2^{x-2}\right) + 1$



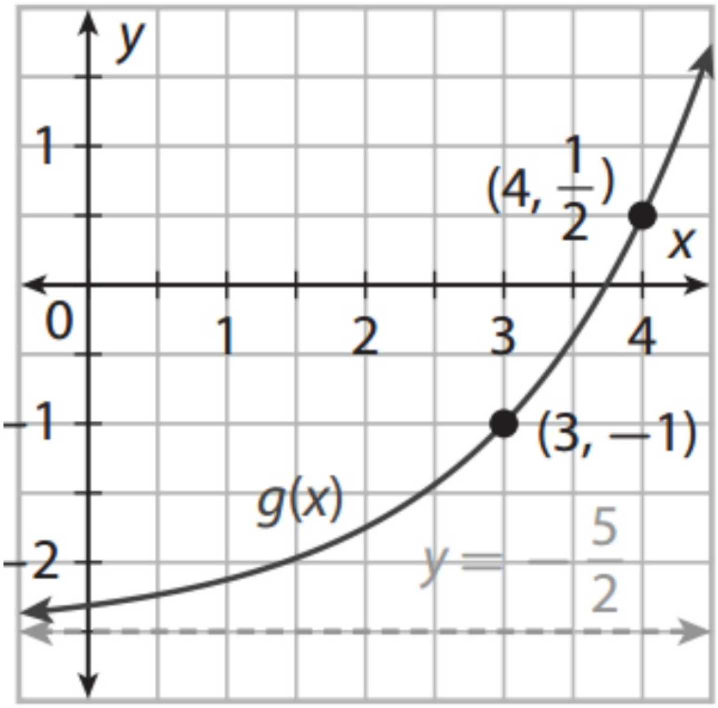
Exponential Models

Writing Equations for Combined Transformations $g(x) = a\left(b^{x-h}\right) + k$

$\left(h, a + k\right)$

$\left(1 + h, ab + k\right)$

$g(x) = \frac{3}{2}\left(2^{x-3}\right) - \frac{5}{2}$



Exponential Models

For exponential growth $y = ab^x$, with $b > 1$, the value b is the **growth factor**.
A quantity that exhibits exponential growth increases by a constant percentage each time period. The percentage increase r, written as a decimal, is the *rate of increase* or *growth rate*. For exponential growth, $b = 1 + r$.

For exponential decay, $0 < b < 1$ and b is the **decay factor**. The quantity decreases by a constant percentage each time period. The percentage decrease, r, is the *rate of decay*. Usually a rate of decay is expressed as a negative quantity, so $b = 1 + r$.

take note

Key Concept Exponential Growth and Decay

You can model exponential growth or decay with this function.

Amount after t time periods

Rate of growth ($r > 0$) or decay ($r < 0$)

$$A(t) = a(1 + r)^t$$

Initial amount

Number of time periods

For growth or decay to be exponential, a quantity changes by a fixed percentage each time period.

Exponential Models

Modeling Exponential Growth

You invested \$1000 in a savings account at the end of 6th grade. The account pays 5% annual interest. How much money will be in the account after six years?

Step 1 Determine if an exponential function is a reasonable model.

The money grows at a fixed rate of 5% per year. An exponential model is appropriate.

Step 2 Define the variables and determine the model.

Let t = the number of years since the money was invested.
Let $A(t)$ = the amount in the account after each year.

A reasonable model is $A(t) = a(1 + r)^t$.

Step 3 Use the model to solve the problem.

The account contains \$1340.10 after six years.

Exponential Models

ANY QUESTIONS

Exponential Models

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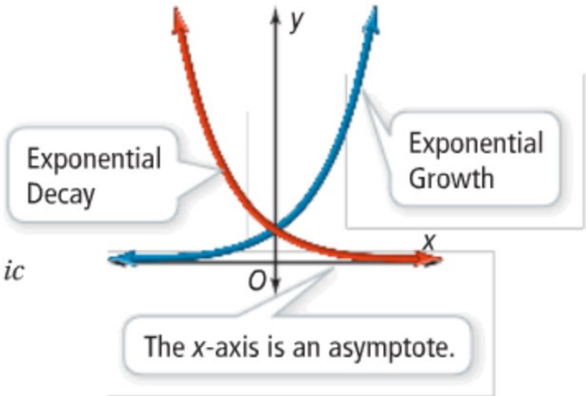

Classwork

Worksheet 13.1

Essential Understanding You can represent repeated multiplication with a function of the form $y = ab^x$ where b is a positive number other than 1.

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